March 1987

UILU-ENG-87-2281
DC-99

COORDINATED SCIENCE LABORATORY
College of Engineering
Decision and Control Laboratory

PARAMETER ADAPTIVE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

David G. Taylor

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION
Unclassified

1b. RESTRICTIVE MARKINGS
None

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION / DOWNGRADING SCHEDULE

3. DISTRIBUTION / AVAILABILITY OF REPORT
Approved for public release; distribution unlimited

4. PERFORMING ORGANIZATION REPORT NUMBER(S)
UILU-ENG-87-2281 (DC-99)

5. MONITORING ORGANIZATION REPORT NUMBER(S)

6a. NAME OF PERFORMING ORGANIZATION
Coordinated Science Lab
University of Illinois

6b. OFFICE SYMBOL *(If applicable)*
N/A

7a. NAME OF MONITORING ORGANIZATION
National Science Foundation

6c. ADDRESS *(City, State, and ZIP Code)*
1101 W. Springfield Avenue
Urbana, IL 61801

7b. ADDRESS *(City, State, and ZIP Code)*
1800 G Street, N.W.
Washington, DC 20550

8a. NAME OF FUNDING / SPONSORING ORGANIZATION
National Science Foundation

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
NSF ECS 87-15811

8c. ADDRESS *(City, State, and ZIP Code)*
1800 G Street, N.W.
Washington, DC 20550

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*
PARAMETER ADAPTIVE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

12. PERSONAL AUTHOR(S) David G. Taylor

13a. TYPE OF REPORT
Technical

13b. TIME COVERED
FROM ________ TO ________

14. DATE OF REPORT *(Year, Month, Day)*
March 1987

15. PAGE COUNT
22

16. SUPPLEMENTARY NOTATION

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
nonlinear systems, adaptive control, feedback control

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

An adaptive state feedback control algorithm is developed for a class of nonlinear systems having unknown parameters which enter the dynamic model in a linear fashion. Provided that certain matchability conditions are satisfied, the tracking error between the nonlinear plant and a linear reference model tends to zero for any feasible initialization of parameter estimates. Synthesis of the controller does not require that the unknown parameters belong to specified intervals of uncertainty. Simulation examples illustrate the application and performance of the control algorithm.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
Unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL

22b. TELEPHONE *(Include Area Code)*

22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

PARAMETER ADAPTIVE CONTROL
FOR A CLASS OF NONLINEAR SYSTEMS

David G. Taylor
Department of Electrical and Computer Engineering
University of Illinois
1406 West Green Street
Urbana, IL 61801
U.S.A.

Abstract. An adaptive state feedback control algorithm is developed for a class of nonlinear systems having unknown parameters which enter the dynamic model in a linear fashion. Provided that certain matchability conditions are satisfied, the tracking error between the nonlinear plant and a linear reference model tends to zero for any feasible initialization of parameter estimates. Synthesis of the controller does not require that the unknown parameters belong to specified intervals of uncertainty. Simulation examples illustrate the application and performance of the control algorithm.

1. Introduction

Many practical control problems involve both nonlinear dynamics and parametric uncertainty in the plant model. The objective is typically to find, if possible, a feedback control which forces the plant state to track the response of a specified linear reference model in spite of the plant nonlinearity and uncertainty. The challenge confronting the control designer is thus to achieve a robust compensation of the plant nonlinearity. Furthermore, engineering considerations may require that this design objective be achieved while satisfying the following criteria: (i) no restrictive bounds on parameter variation may be assumed for controller synthesis; (ii) the model following property must be achieved in a global sense.

Consider the uncertain nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where $f(x)$ and $g(x)$ are supposed to be C^∞ vector fields on R^n , $f(0) = 0$, $g(x)$ is nonzero on R^n , and $u \varepsilon R$ is the control input. In (1), $f(x)$ and $g(x)$ are not known precisely but instead are assumed to be characterized as

$$\begin{aligned} f(x) &\triangleq \hat{f}(x) + \Delta f(x) \\ g(x) &\triangleq \hat{g}(x) + \Delta g(x) \end{aligned} \tag{2}$$

where $\hat{f}(x)$ and $\hat{g}(x)$ represent the available model of the plant, while $\Delta f(x)$ and $\Delta g(x)$ denote the modeling error which is due to a lack of knowledge regarding some plant parameters. The linear reference model, which indicates the desired behavior for the plant, is taken to be

$$\dot{x}_r = Ax_r + bv \tag{3}$$

where $A \varepsilon R^{nxn}$ is a Hurwitz matrix, $b \varepsilon R^n$ and (A,b) is a controllable pair. The state tracking error

$$e = x_r - x \tag{4}$$

represents the measured deviation of the plant state from the reference model trajectory.

The control problem addressed in this paper is described as follows. "Given $\hat{f}(x)$, $\hat{g}(x)$, A and b, determine if possible a state feedback control algorithm such that $e(t) \to 0$ as $t \to \infty$." The solution to this type of design problem involves two distinct parts (see, for instance, [1,2]). The first subject is concerned with the structural compatibility between the nonlinear plant and the linear reference model. If certain matchability conditions are met, then a static state feedback controller is guaranteed to exist such that the model following property holds. Of course, due to parameter uncertainty the existence of a model following controller does not provide the required realization. Hence, the second subject concerns the design of an adaptive control algorithm which assures that perfect model following is indeed achieved.

2. System Structure

The class of systems for which linear model following controls exist has been a topic of recent research [3,4,5]. The existence of such a controller is assured whenever the nonlinear plant is feedback equivalent to a linear controllable system [3]. Our goal is to develop a controller which solves the problem, and we choose here to explore the case when no coordinate changes in the state space are allowed.

We shall require the following assumptions:

(A1) The actual plant is parameterized according to

$$\begin{aligned} f(x) &= f_0(x) + F(x)p_F \\ g(x) &= g_0(x) + G(x)p_G \end{aligned} \tag{5}$$

where $f_0(x) \in R^n$, $g_0(x) \in R^n$, $F(x) \in R^{nxr}$ and $G(x) \in R^{nxs}$ are known, whereas p_F and p_G are uncertain parameter vectors in R^r and R^s, respectively. The vector fields $f_0(x)$ and $g_0(x)$ are included in case not all plant parameters are unknown. The available plant model is likewise taken to be

$$\begin{aligned} \hat{f}(x) &= f_0(x) + F(x)\,\hat{p}_F \\ \hat{g}(x) &= g_0(x) + G(x)\,\hat{p}_G \end{aligned} \tag{6}$$

where $\hat{p}_F$ and $\hat{p}_G$ represent the estimates of the true parameter vectors defined in (5);

(A2) Matching conditions are satisfied by the uncertainty due to plant modeling error

$$\begin{aligned} \Delta f(x) &\in \hat{\mathcal{G}}(x) \\ \Delta g(x) &\in \hat{\mathcal{G}}(x) \end{aligned} \tag{7}$$

and the choice of linear reference model

$$\begin{aligned} Ax - \hat{f}(x) &\in \hat{\mathcal{G}}(x) \\ b &\in \hat{\mathcal{G}}(x) \end{aligned} \tag{8}$$

for all $x \in R^n$ where $\hat{\mathcal{G}}(x)$ denotes the distribution

$$\hat{\mathcal{G}}(x) = \text{span}\,\{\hat{g}(x)\} \tag{9}$$

associated with the available plant model;

(A3) The sign, and lower bound on magnitude, is known for each component of vector p_G.

The main assumption is (A1), invoked for reasons unique to our method of solution. It plays a crucial role in providing the desired error dynamics. The assumption (A2) is a basic one which indicates the variety of structured parametric uncertainty we allow and assures that linear model following is an achievable goal. The remaining assumption (A3) will aid in maintaining the estimate $\hat{p}_G$ in some feasible region of R^s, but will rarely be required.

Note that (A2) implies the existence of functions $\alpha(x)$ and $\beta(x)$, with $\alpha(0) = 0$ and $\beta(x) \neq 0$ in R^n, such that

$$
\begin{aligned}
Ax - f(x) &= g(x)\alpha(x) \\
b &= g(x)\beta(x)
\end{aligned}
\tag{10}
$$

According to (10), the tracking error dynamics resulting from the feedback control

$$
u = \alpha(x) + \beta(x)v \tag{11}
$$

are computed to be

$$
\dot{e} = Ae \tag{12}
$$

and hence $e(t) \to 0$ as $t \to \infty$. However, there is no constructive procedure for obtaining this model following control law even when it is known to exist. Instead, we are obliged to use the available information to choose functions $\hat{\alpha}(x)$ and $\hat{\beta}(x)$ for the control satisfying

$$
\begin{aligned}
Ax - \hat{f}(x) &= \hat{g}(x)\hat{\alpha}(x) \\
b &= \hat{g}(x)\hat{\beta}(x)
\end{aligned}
\tag{13}
$$

Such functions are guaranteed to exist as a consequence of (A2) and a solution of (13) is

$$
\begin{aligned}
\hat{\alpha}(x) &= \hat{g}^{\dagger}(x)\ \left[Ax - \hat{f}(x)\right] \\
\hat{\beta}(x) &= \hat{g}^{\dagger}(x)\ b
\end{aligned}
\tag{14}
$$

where

$$
\hat{g}^{\dagger}(x) \triangleq \left[\hat{g}^{T}(x)\ \hat{g}(x)\right]^{-1} \hat{g}^{T}(x)
$$

denotes the pseudo-inverse of $\hat{g}(x)$. The following result will prove useful for the stability analysis of the adaptive control system.

Lemma. Suppose that (A1) and (A2) are satisfied. If the feedback control

$$u(x,v) = \hat{\alpha}(x) + \hat{\beta}(x)v \tag{15}$$

defined by (14) is applied to system (1), then the resulting tracking error satisfies the differential equation

$$\dot{e} = Ae - \Phi^T(x,v)\theta \tag{16}$$

where

$$\Phi^T(x,v) \overset{\Delta}{=} [F(x), \quad G(x)u(x,v)]_{nx(r+s)}$$

$$\theta \overset{\Delta}{=} \begin{bmatrix} p_F - \hat{p}_F \\ p_G - \hat{p}_G \end{bmatrix}_{(r+s)x1}$$

Proof. In general, the plant dynamics are

$$\dot{x} = (\hat{f}(x) + \Delta f(x)) + (\hat{g}(x) + \Delta g(x))u \tag{17}$$

and hence the control (15), chosen to satisfy (13), induces error dynamics

$$\dot{e} = Ae - \Delta f(x) - \Delta g(x)\ (\hat{\alpha}(x) + \hat{\beta}(x)v) \tag{18}$$

According to (5) and (6), the selected parameterization corresponds to perturbations given by

$$\begin{aligned} \Delta f(x) &= F(x)(p_F - \hat{p}_F) \\ \Delta g(x) &= G(x)(p_G - \hat{p}_G) \end{aligned} \tag{19}$$

Using (19), system (18) may be expressed in the form of (16) with the appropriate interpretations of $\Phi^T(x,v)$ and θ. $\square$

Note that when $\theta \equiv 0$ then by hypothesis $\hat{\alpha}(x) \equiv \alpha(x)$ and $\hat{\beta}(x) \equiv \beta(x)$, in which case the tracking error dynamics (16) agree with (12). When parameter errors do exist, however, the Lemma clarifies the effects of using $\hat{\alpha}(x)$ and and $\hat{\beta}(x)$ instead of $\alpha(x)$ and $\beta(x)$ in the control law. The perturbation term $\phi^T(x,v)\theta$ is due to the imperfect nonlinearity compensation achieved by feedback control (15). In spite of the nonlinearity present in this problem, we shall show that an adaptive algorithm may be designed by the same Lyapunov methods common to earlier results [6,7] in adaptive control of unknown linear systems.

3. Adaptive Control Algorithm

In this section, a solution to the control problem posed in Section 1 is presented subject to assumptions (A1)-(A3). Under more restrictive assumptions the problem may be dealt with by existing techniques related to the stabilization of uncertain nonlinear systems. One approach [8] makes use of static feedback linearization together with an additional control loop designed to counteract the destabilizing effect of imperfect nonlinearity compensation. Robustness is assured in these continuous feedback designs by including, for example, a linear high gain controller [9] or a nonlinear saturating controller [10]. These methods require knowledge of sufficiently small bounding sets for the uncertainty. Another approach [11,12] involves the use of discontinuous control in a model following scheme based on either variable structure methods or a hyperstability approach. No direct attempt at cancelling plant nonlinearity is made, but the influence of the switching control achieves the linearization in an approximate sense. Again, the controller synthesis requires knowledge of the range of variation of all plant parameters. For a specific class of nonlinear systems, rigid robot manipulators with unknown parameters, the restrictive modeling assumptions common to the methods just described have been removed by using a parameter adaptive control [13,14]. Global stability is achieved but the problem formulation and solution are application specific. Hence, apparently none of these available design methodologies solves the problem of global model following subject to (A1)-(A3).

In this paper, we follow a parameter adaptive approach. Given some initial guess for the parameters, say $\hat{p}_F(0)$ and $\hat{p}_G(0)$, a parameter update law governs the evolution of the parameter estimates, $\hat{p}_F(t)$ and $\hat{p}_G(t)$, according to the measured tracking error $e(t)$. Specifically, we consider

$$\begin{aligned} \dot{\hat{p}}_F(t) &= F^T(x)\, P\, e(t) \\ \dot{\hat{p}}_G(t) &= G^T(x)\, P\, e(t)u(t,x,v) \end{aligned} \tag{20}$$

where the nxn matrix P is the unique solution of the Lyapunov equation

$$A^TP + PA = -Q \tag{21}$$

for any positive definite and symmetric nxn matrix Q. These parameter estimates subsequently provide estimates of the vector fields

$$\begin{aligned} \hat{f}(t,x) &= f_0(x) + F(x)\hat{p}_F(t) \\ \hat{g}(t,x) &= g_0(x) + G(x)\hat{p}_G(t) \end{aligned} \tag{22}$$

The control algorithm is taken to be the "certainty equivalence" form of that described by the Lemma, that is to say

$$u(t,x,v) = \hat{\alpha}(t,x) + \hat{\beta}(t,x)v \tag{23}$$

where

$$\begin{aligned} \hat{\alpha}(t,x) &= \hat{g}^{\dagger}(t,x)\,[Ax - \hat{f}(t,x)] \\ \hat{\beta}(t,x) &= \hat{g}^{\dagger}(t,x)b \end{aligned}$$

In keeping with the notation of the Lemma, we define

$$\Phi^T(t,x,v) = [F(x),\ G(x)u(t,x,v)]$$

$$\theta(t) = \begin{bmatrix} p_F - \hat{p}_F(t) \\ p_G - \hat{p}_G(t) \end{bmatrix} \tag{24}$$

and from (24) the update law (20) may be equivalently expressed as

$$\dot{\theta}(t) = \Phi(t,x,v)P\,e(t) \tag{25}$$

Technical difficulties arise if, for any $\tau \geq 0$, the adaptation mechanism (25) influences $\hat{g}(t,x)$ in such a way that $\hat{g}^\dagger(\tau,x)$ does not exist. Note in this case that the control law (23) is not well-defined at $t = \tau$. Such an occurence may be avoided by virtue of (A3) using a "constrained" update [2,14] for $\hat{p}_G(t)$ so that $\hat{p}_G(t) \neq 0$ for all $t \geq 0$. Of course, if all uncertainty belongs to f(x) then this procedure is not needed and (A3) may be disregarded as well. A block diagram of the adaptive control system is shown in Fig. 1.

<u>Theorem.</u> Suppose assumptions (A1) and (A2) are satisfied, and consider the adaptive control algorithm (23) with parameter update law (25). If $\hat{p}_G(t) \neq 0$ for all $t \geq 0$ and reference signal v(t) is uniformly bounded, then the equilibrium point $(e,\theta) = 0$ of the closed-loop system is uniformly stable over $[0,\infty)$ and $e(t) \to 0$ as $t \to \infty$.

<u>Proof.</u> If $\hat{p}_G(t) \neq 0$, then the control algorithm is well-defined and the dynamics of the (n+r+s) order closed-loop system in error coordinates are

$$\begin{aligned} \dot{e} &= Ae - \Phi^T\theta \\ \dot{\theta} &= \Phi P e \end{aligned} \tag{26}$$

where arguments are suppressed for brevity. The origin of the nonautonomous error system (26) is an equilibrium point for all $t_0 \varepsilon [0,\infty)$. We choose the candidate Lyapunov function

$$V(e,\theta) = e^T P e + \theta^T \theta \tag{27}$$

Note that (27) is a decrescent positive definite function since P, according to (21), is a symmetric positive definite matrix. The time derivative of (27) evaluated along trajectories of (26) is

$$\dot{V}(e,\theta) = e^T(A^T P + PA)e + 2\theta^T(\dot{\theta} - \Phi Pe) \tag{28}$$

Substituting (21) and (25) into (28) yields

$$\dot{V}(e,\theta) = -e^T Q e \leqslant 0 \tag{29}$$

where, by hypothesis, Q is a symmetric positive definite matrix. Hence, the origin is uniformly stable over $[0,\infty)$ and $e(t)$ and $\theta(t)$ are uniformly bounded. Since $v(t)$ is uniformly bounded, then from (26), $\dot{e}(t)$ and $\dot{\theta}(t)$ are uniformly bounded also. This implies that $e(t)$, $\theta(t)$, $V(e,\theta)$ and $\dot{V}(e,\theta)$ are all uniformly continuous functions of time. Since $V(e,\theta)$ is nonincreasing (29) and bounded below (27), $V(e,\theta)$ has a definite limit as $t \to \infty$. Denoting this limit by V_∞, it follows that

$$\lim_{t\to\infty} \int_0^t \dot{V}(e,\theta)dt = V_\infty - V(e(0),\ \theta(0)) < \infty \tag{30}$$

Since $\dot{V}(e,\theta)$ is nonpositive with finite integral, from (30) we conclude

$$\lim_{t\to\infty} \dot{V}(e,\theta) = 0 \tag{31}$$

Together, (29) and (31) imply that

$$\lim_{t\to\infty} e(t) = 0 \tag{32}$$

and the Theorem is proved. $\square$

To guard against the remote possibility of $\hat{p}_G(t)$ vanishing when $\Delta g(x) \neq 0$, one approach is to find a modification of (25) which assures that no sign changes take place for any component of $\hat{p}_G(t)$. We define the feasible set

$$\Omega = \{\hat{p}_G \epsilon R^s: \ \mathrm{sgn}(\hat{p}_{Gi}) = \mathrm{sgn}(p_{Gi}), \left|\hat{p}_{Gi}\right| \geq \sigma_i > 0, \ i = 1,\ldots,s\} \tag{33}$$

where according to (A3), $\mathrm{sgn}(p_{Gi})$ and σ_i are known for $i = 1,\ldots,s$. The set $\Omega \subset R^s$ is a convex hypercube and clearly $p_G \ \epsilon\ \Omega$. If the estimate $\hat{p}_G(t)$ reaches the boundary of the feasible set Ω at $t = \tau$, then (25) is modified componentwise according to the logic

$$\hat{p}_{Gi}(\tau^+) = \begin{cases} \sigma_i \ \mathrm{sgn}(p_{Gi}) & , \ \text{if} \left|\hat{p}_{Gi}(\tau)\right| \leq \sigma_i - \delta_i \\ \hat{p}_{Gi}(\tau) & , \ \text{otherwise} \end{cases} \tag{34}$$

where δ_i are small positive design parameters satisfying $\delta_i < \sigma_i$ for $i = 1,\ldots,s$. Each time this modification is activated, the basic update law (25) is reinitialized with a feasible estimate. As a consequence, if $\hat{p}_G(0) \ \epsilon\ \Omega$ then $\hat{p}_G(t) \neq 0$ for all $t \geq 0$, thus removing any remote possibility of singularity in the control law (23).

Suppose that at time $t = \tau$, the logic (34) re-initializes the parameter estimate $\hat{p}_G(\tau)$. It suffices to show that at $t = \tau$ the function $V(e,\theta)$ retains its nonincreasing property in spite of this modification of the primary update law (25). Generally speaking, any number of components of $\hat{p}_G(\tau)$ may be reset. Let these components be identified by the set $I \subset \{1,\ldots,s\}$ such that $i \epsilon I$ implies $\hat{p}_{Gi}(\tau)$ requires modification. From (34) and (27) we compute

$$V(e(\tau^+),\ \theta(\tau^+)) = V(e(\tau),\ \theta(\tau)) - \sum_{i\varepsilon I} [2\delta_i(|p_{Gi}| - \sigma_i) + \delta_i^{\ 2}] \tag{35}$$

Hence, we conclude from (34) and (35) that at $t = \tau^+$

$$0 \leq V(e(\tau^+),\ \theta(\tau^+)) < V(e(\tau),\ \theta(\tau)) \tag{36}$$

According to (35) and (36), for all $t\varepsilon\ [0,\infty)$ $V(e,\theta)$ is nonincreasing, bounded below by 0, and decremented by a strictly positive quantity no less than $\delta_i^{\ 2}$ at times $t = \tau_j$, $j \geq 1$. Consequently, the modification logic will be activated at most a finite number of times. It follows that the trajectory of the system will remain bounded and that, after some finite length of time, the dynamics of the system will be exactly described by (26) with no further modifications. Therefore, the previous analysis is eventually valid.

Remark 1: According to the Theorem, the adaptive control algorithm (23), (25), (34) achieves the desired result. Given any feasible initial guess of plant parameters ($\hat{p}_F(0)\varepsilon R^r$, $\hat{p}_G(0)\varepsilon\Omega\subset R^s$), the tracking error $e(t) \to 0$ as $t \to \infty$, for any $e(0)\ \varepsilon R^n$.

Remark 2: The error dynamics (26) indicate that if $\theta(t) \to 0$, then subsequently $e(t) \to 0$ as $t \to \infty$ as well. However, it is not necessary for the parameter estimates to converge to their true values in order that $e(t) \to 0$ as $t \to \infty$.

Remark 3: Suppose that, in addition to satisfying the hypotheses of the Theorem, there exist positive numbers T, ε and δ such that for all $t_0 \geq 0$ and for any unit vector $w\varepsilon R^{r+s}$, there is a $t\varepsilon\ [t_0, t_0 + T]$ such that

$$\left\| \int_t^{t+\delta} \Phi^T(\tau, x(\tau), v(\tau))w\ d\tau \right\| \geq \varepsilon \tag{37}$$

Then from [15] we may conclude that the origin of error system (26) is uniformly asymptotically stable. In other words, if the reference trajectory v(t) is selected such that (37) is satisfied, then both e(t) and $\theta(t)$ of system (26) will converge to zero.

4. Illustrative Examples

Example 1. We recall from [16] the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a x_2^3 + u \end{aligned} \tag{38}$$

where a is an unknown constant parameter. In [16] two feedback designs are presented, the goal of each being state regulation to the origin. The first design, based on the linear approximation of (38) at the origin, employs the linear high-gain feedback

$$u = -\gamma^2 x_1 - \gamma x_2 \tag{39}$$

where γ is a positive design parameter. Without knowledge of the sign of a, only local stability of the origin can be deduced. If $a > 0$ then any initial conditions belonging to the set

$$S_1 = \{x \varepsilon R^2 :\ \gamma x_1^{\ 2} + \frac{1}{\gamma} x_2^{\ 2} > a^{-2}\} \tag{40}$$

will result in divergent unbounded state trajectories. The technique of static feedback linearization is the basis for the second design

$$u = -\hat{a}\, x_2^3 - \gamma^2 x_1 - \gamma x_2 \tag{41}$$

where $\hat{a}$ is the modeled value of a. This design achieves perfect nonlinearity compensation only if $\hat{a} = a$. If a is underestimated ($\hat{a} < a$), then all trajectories with initial conditions contained in

$$S_2 = \{x \varepsilon R^2 : \gamma x_1^{\ 2} + \frac{1}{\gamma} x_2^{\ 2} > (a-\hat{a})^{-2}\} \tag{42}$$

will diverge to infinity.

According to the discussion above, two popular design methodologies lead to dissatisfying results when applied to system (38). The global stability of the origin is not robust with respect to the unknown parameter a. Moreover, (40) and (42) indicate that the stability regions around $x = 0$ vanish as $\gamma \to \infty$. Note, however, that with reference model

$$\dot{x}_r = \begin{bmatrix} 0 & 1 \\ -\gamma^2 & -\gamma \end{bmatrix} x_r \tag{43}$$

assumptions (A1) and (A2) are satisfied ((A3) is not needed here). Using the adaptive control

$$\begin{aligned} u(t,x) &= -\hat{a}(t)x_2^{\ 3} - \gamma^2 x_1 - \gamma x_2 \\ \dot{\hat{a}}(t) &= -kx_2^{\ 3}\Gamma e \qquad , \ \Gamma = [\frac{1}{\gamma^2} \ \frac{1+\gamma^2}{\gamma^3}] \end{aligned} \tag{44}$$

the feedback system becomes third order and the origin of error coordinates (e,θ) is uniformly stable over $[0,\infty)$. Moreover, for any $\hat{a}(0)\varepsilon R$ and for all $x(0)\varepsilon R^2$, we are assured that $x(t) \to 0$ as $t \to \infty$.

This global regulation result is further illustrated in the following simulation. Suppose that $a = \frac{1}{3}$ but our best estimate of a is $\hat{a} = \frac{1}{4}$. Suppose also that the performance objective is consistent with the choice $\gamma = 5$ and that the initial data is $x(0) = x_r(0) = (6,6)$. It is easily verified from (40) and (42) that either control (39) or (41) results in unbounded trajectories. A simulation with the adaptive controller (44) is shown in Figs. 2(a), (b) and (c), where the adaptation gain is $k = 5 \times 10^{-4}$. Note that although the parameter error $\theta(t)$ has a nonzero steady-state value, the tracking error $e(t)$ vanishes and hence $x(t) \to 0$ as $t \to \infty$.

Example 2. Consider the nonlinear system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= a \sin x_1 + bu \end{aligned} \tag{45}$$

where a and b are unknown constant parameters with $b \geq 1$. The pair (x_1, x_2) represents the position and velocity of a single link robot manipulator with no joint flexibility. The dynamics of the actuator have been assumed to be negligibly fast so that the input u is the joint torque. The objective is to find a feedback control which forces the state of (45) to track a path prescribed as the response of the system

$$\dot{x}_r = \begin{bmatrix} 0 & 1 \\ -\gamma_1 & -\gamma_2 \end{bmatrix} x_r + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v \tag{46}$$

to the reference signal v, so that $x(t) \to x_r(t)$ as $t \to \infty$.

The assumptions (A1)-(A3) are satisfied. Hence, consider now the adaptive controller

$$u(t,x,v) = \hat{b}(t)^{-1}[-\hat{a}(t) \sin x_1 - \gamma_1 x_1 - \gamma_2 x_2 + v]$$

$$\dot{\hat{a}}(t) = - k \sin x_1 \Gamma e$$

$$\dot{\hat{b}}(t) = -\,ku(t,x,v)\Gamma e$$
$$\Gamma = \left[\frac{1}{\gamma_1} \quad \frac{1+\gamma_1}{\gamma_1\gamma_2}\right] \tag{47}$$

where $\hat{b}(\tau)$ is set equal to 1 if $|\hat{b}(\tau)| < 1 - \delta$ where $\delta\varepsilon(0,1)$. By virtue of the Theorem, the origin of the fourth order error system is uniformly stable over $[0,\infty)$. Furthermore, for any $\hat{a}(0)\varepsilon R$, $\hat{b}(0)\varepsilon[1,\infty)$ and $e(0)\ \varepsilon R^2$, perfect path tracking is achieved since $x(t) \to x_r(t)$ as $t \to \infty$.

In the following simulation, we assume that the true parameters are $a = 10$ and $b = 2$, whereas the available initial estimates are $\hat{a} = 5$ and $\hat{b} = 4$. The reference model gains are $\gamma_1 = 16$ and $\gamma_2 = 8$ for a critically damped response to a step function $v = 16$, and the initial conditions on state variables are $x(0) = x_r(0) = 0$. If the adaptation is disabled by setting $k = 0$ in (47), then the path tracking error $e_1(t)$ is large with a steady-state value of -0.9. However, it is apparent from Figs. 3(a) and (b) that the use of adaptation with $k = 57$ results in a superior response. It is clear from Figs. 3(c) and 3(d) that although $e(t) \to 0$ as $t \to \infty$, convergence of the parameter estimates is not achieved. In this simulation, the parameter $\hat{b}(t)$ evolves in R without reaching the specified lower bound on magnitude.

5. Conclusion

A parameter adaptive control algorithm has been developed for a class of nonlinear systems. Under certain conditions, the closed-loop system is uniformly stable and the tracking error tends to zero. The controller design is based on Lyapunov methods and requires full state measurement. For nonlinear systems with unknown parameters, the adaptive control algorithm presents advantages over static nonlinear feedback from the viewpoints of stability and performance.

Acknowledgement

The author is grateful for many beneficial discussions with Professor Petar Kokotovic.

References

[1] Y. D. Landau, Adaptive Control: The Model Reference Approach. Marcel Dekker: New York, NY, 1979.

[2] G. C. Goodwin and K. S. Sin, Adaptive Filtering, Prediction, and Control. Prentice-Hall: Englewood Cliffs, NJ, 1984.

[3] R. Marino and S. Nicosia, "Linear model following control and feedback equivalence to linear controllable systems," International Journal of Control, vol. 39, no. 3, pp. 473-485, 1984.

[4] R. Su, "On the linear equivalents of nonlinear systems," Systems and Control Letters, vol. 2, no. 1, pp. 48-52, 1982.

[5] L. R. Hunt, R. Su and G. Meyer, "Global transformations of nonlinear systems," IEEE Transactionson Automatic Control, vol. AC-28, no. 1, pp. 24-31, 1983.

[6] K. S. Narendra and L. S. Valavani, "Stable adaptive controller design: direct control," IEEE Transactions on Automatic Control, vol. AC-23, no. 4, pp. 570-583, 1978.

[7] K. S. Narendra, Y. H. Lin and L. S. Valavani, "Stable adaptive controller design, part II: proof of stability," IEEE Transactions on Automatic Control, vol. AC-25, no. 3, pp. 440-448, 1980.

[8] M. W. Spong, "Robust stabilization for a class of nonlinear systems," Theory and Applications of Nonlinear Control Systems. North Holland: New York, NY, pp. 155-165, 1986.

[9] J. S. Thorp and B. R. Barmish, "On guaranteed stability of uncertain linear systems via linear control," Journal of Optimization Theory and Applications, vol. 35, no. 4, pp. 559-579, 1981.

[10] G. Leitmann, "On the efficacy of nonlinear control in uncertain linear systems," ASME Journal of Dynamic Systems, Measurement, and Control, vol. 103, no. 2, pp. 95-102, 1981.

[11] A. Balestrino, G. DeMaria and A.S.I. Zinober, "Nonlinear adaptive model following control," Automatica, vol. 20, no. 5, pp. 559-568, 1984.

[12] A. Balestrino, G. DeMaria and L. Sciavicco, "Hyperstable adaptive model following control of nonlinear plants," Systems and Control Letters, vol. 1, no. 4, pp. 232-236, 1982.

[13] R. H. Middleton and G. C. Goodwin, "Adaptive computed torque control for rigid link manipulators," Proc. 25th IEEE Conf. Decision and Control, pp. 68-73, December 1986.

[14] J. J. Craig, P. Hsu and S. S. Sastry, "Adaptive control of mechanical manipulators," Proc. IEEE Conf. Robotics and Automation, pp. 190-195, April 1986.

[15] A. P. Morgan and K. S. Narendra, "On the stability of nonautonomous differential equations $\dot{x} = [A+B(t)]x$, with skew symmetric matrix $B(t)$," SIAM Journal of Control and Optimization, vol. 15, no. 1, pp. 163-176, 1977.

[16] P. V. Kokotovic and R. Marino, "On vanishing stability regions in nonlinear systems with high-gain feedback," IEEE Transactions on Automatic Control, vol. AC-31, no. 10, pp. 967-970, 1986.

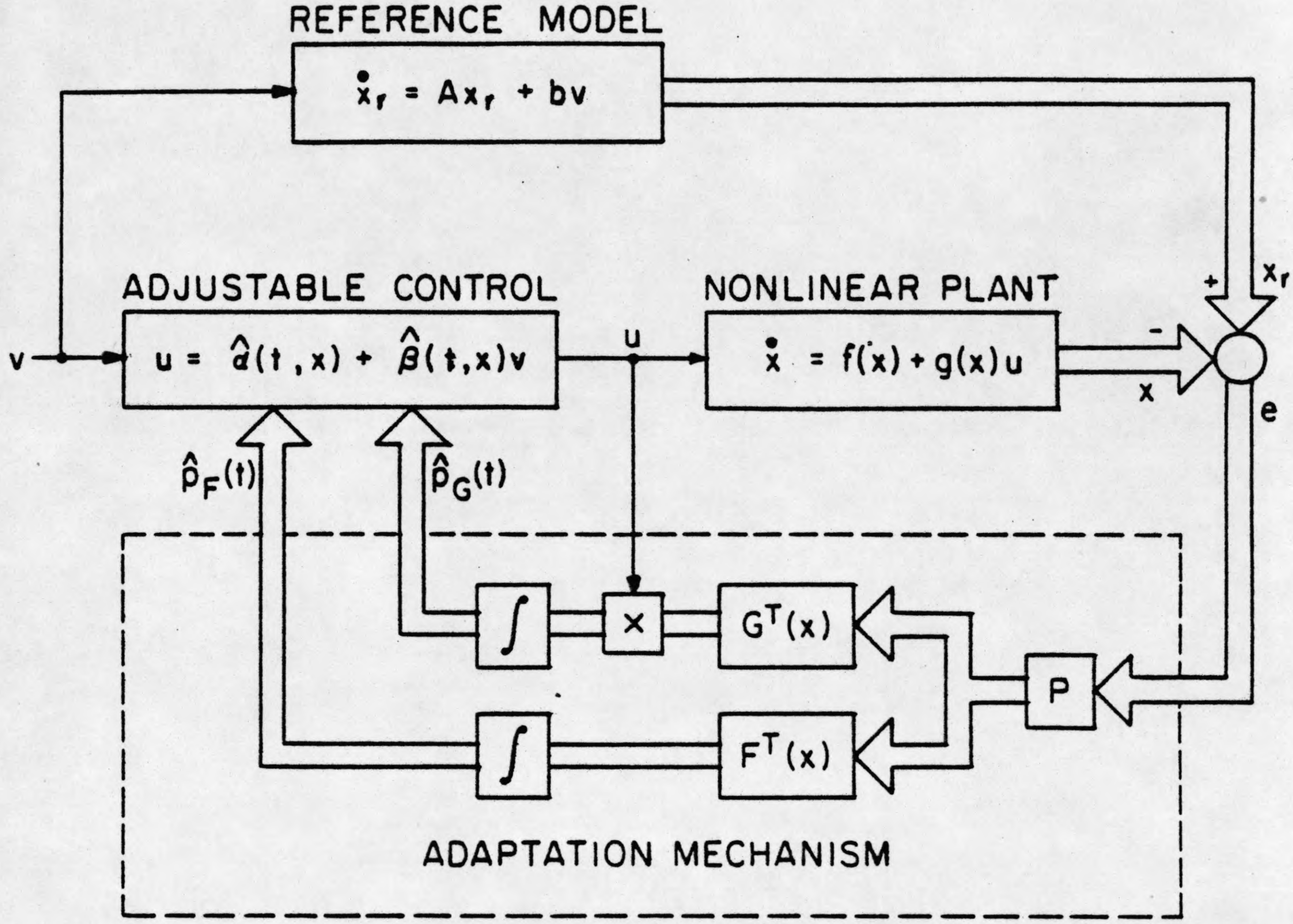

Fig. 1. Block diagram of the adaptive control system.

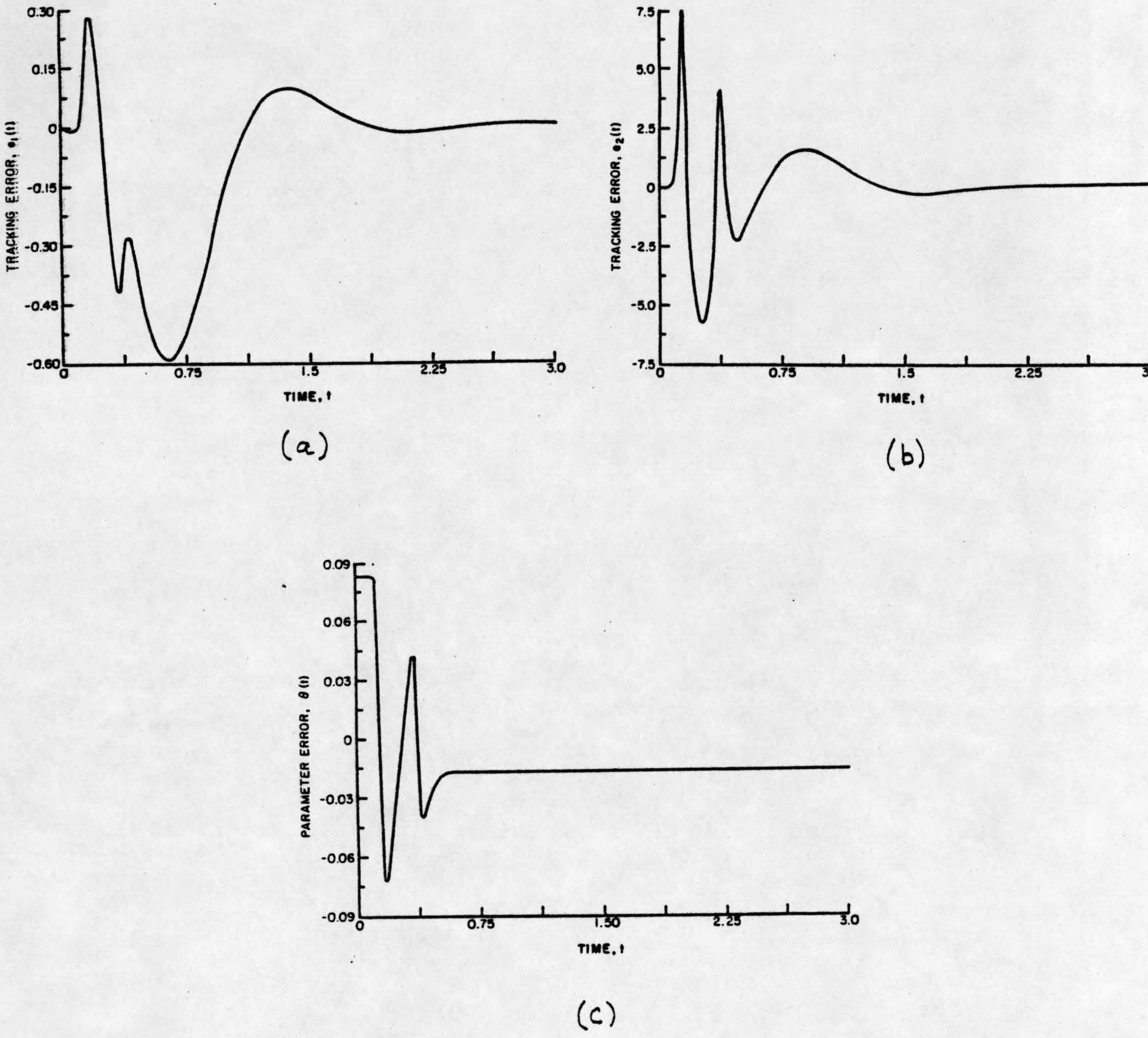

Fig. 2. Simulated response for example 1. (a) Tracking error e_1. (b) Tracking error e_2. (c) Parameter error θ.

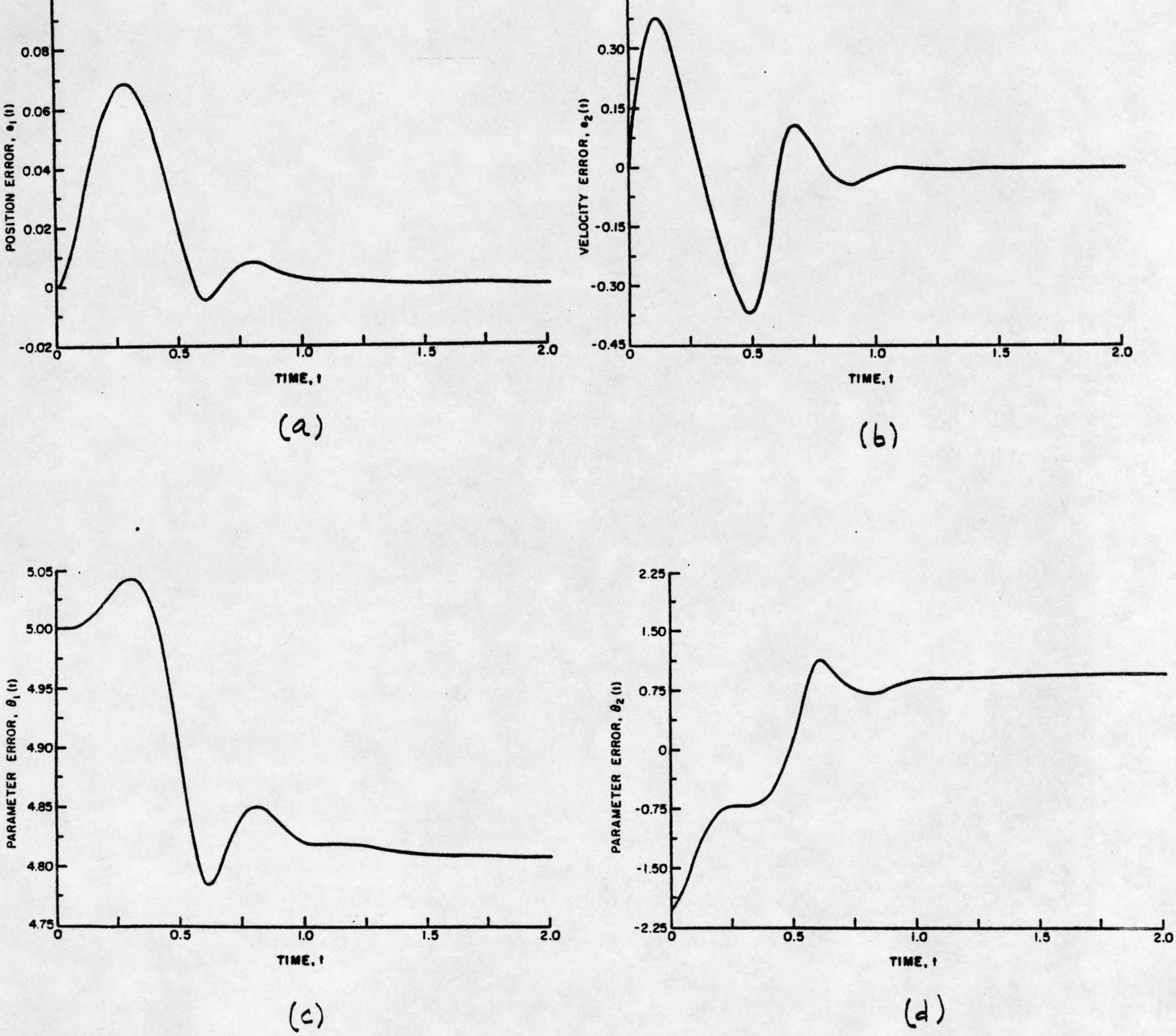

Fig. 3. Simulated response for example 2. (a) Tracking error e_1. (b) Tracking error e_2. (c) Parameter error Θ_1. (d) Parameter error Θ_2.